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Assignment: Section 5.1: 6 (see definition of n! pg. 151), 8, 14, 18, 28, 38, 40 ( 7th Edition)

6.

Using mathematical induction in the basis step, for n = 1, the equation states that 1 · 1! = (1 + 1)! − 1, and this is true because both sides of the equation evaluate to 1. For the inductive step, we assume that 1 · 1! + 2 · 2! + … + k · k! = (k + 1)! − 1 for some positive integer k. We add (k + 1)(k + 1)! to the left hand side to find that 1 · 1! + 2 · 2! + · · · + (k + 1) · (k + 1)! = (k + 1)! − 1 + (k + 1)(k + 1)! The right hand side equals (k + 1)!(k + 2) − 1 = (k + 2)! − 1. This establishes the desired equation also for k + 1, and we are done by the principle of mathematical induction.

8.

Base Case: P(0) is that 2(−7)0 = 2 = (1 − (−7))/4 = 8/4.

Assume that P(n) is true

Inductive Step: We show P(n + 1) is true.

2 − 2 · 7 + ... + 2(−7)n + 2(−7)n+1 = (1 − (−7)^n+1)/4 + 2(−7)^n+1

= (1 − (−7)^n+1 + 8(−7)^n+1)/4

= (1 − (1 − 8)(−7)^n+1)/4

= (1 − (−7)^n+2)/4

14.

We will show this by induction on n.

Base: Consider when n = 1: \sum1 k=1 k2^k = 1 · 2^1 = 2 and (1 − 1)2^1+1 + 2 = 2. Since these are equal, the formula holds for n = 1.

Induction: Suppose that the claim holds for n = i. That is, there is an i ∈ Z^+ such that \sumi k=1 k2^k = (i−1)2^i+1 +2. We need to show: \sumi+1 k=1 k2^k = ((i+1)−1)2(i+1)+1 +2 = i2^i+2 +2 By pulling a term out of the summation, we can write \sumi+1 k=1 k2^k as \sumi k=1 k2^k + (i + 1)2^i+1 . Now we can substitute the k case from the induction hypothesis:

\sum i k=1 k2^k + (i + 1)2^i+1 = (i − 1)2^i+1 + 2 + (i + 1)2^i+1

= i · 2^i+1 – 2^i+1 + 2 + i · 2^i+1 + 2^i+1

= i(2^i+1 + 2^i+1) + 2

= i(2 · 2^i+1) + 2

= i2^i+2 + 2

18.

a) P(2) is 2! < 2^2.

b) True because 2 < 4

c) P(k) is k! < k^k

d) You need to show that assuming the inductive hypothesis(part c), we can show (k + 1)! < (k + 1)^k+1.

e) Multiply (k + 1) to both sides of the inequality asserted by P(k). Here we have:

k! **∙** (k + 1) < k^k **∙** (k + 1) < (k + 1)^k **∙** (k + 1) = (k + 1)^k+1

f)Since the basis and inductive step are completed, using the principle of mathematical induction, this statement is true for every integer greater than 1.

28.

Basis: Let n = 3. Then n 2 − 7n + 12 = 32 − 7 · 3 + 12 = 9 − 21 + 12 = 0.

Inductive hypothesis: Assume for some integer k ≥ 3 that k 2 − 7k + 12 is nonnegative. Inductive step:

(k + 1)^2 − 7(k + 1) + 12 = k^2 + 2k + 1 − 7k − 7 + 12

= (k^2 − 7k + 12) + (2k + 1 − 7) ≥ 0 + 2k + 1 − 7

= 2k − 6 ≥ 2 · 3 − 6

= 0

38.

Basis: P(1) asserts that A1 \subseteq \!\, B1, which directly implies that 1 ∪j=1 Aj \subseteq \!\, 1∪j=1 Bj.

Inductive: The inductive hypothesis is the statement that P(k) is true, where k is a positive integer. That is, P(k) is the statement that if Aj \subseteq \!\, Bj for j = 1, 2,…, k, then k+1 ∪ j=1 Aj \subseteq \!\, k+1 ∪ j=1 Bj. If x is an element of k+1 ∪ j=1 Aj then we can anticipate that x [Є](http://en.wikipedia.org/wiki/Ukrainian_Ye) A **∨** k+1. If x [Є](http://en.wikipedia.org/wiki/Ukrainian_Ye) A k+1, we know from the given fact that A k+1 \subseteq \!\, Bk+1 that x [Є](http://en.wikipedia.org/wiki/Ukrainian_Ye) Bk+1. Therefore, we have shown that if the inductive hypothesis P(k) is true, then P(k + 1) must also be true. This completes the inductive argument. So based on the basis step and the inductive step, we have used mathematical induction to prove P(n) is true for all positive integers.

40.

(A1 ∩ A2) U B = (A1 U B) ∩ (A2 U B)........distributive Property of union over intersection   
  
Assume that the statement is true for k, where k > 2, then:   
  
(A1 ∩ A2 ∩ · · · ∩ Ak+1) U B = ((A1 ∩ A2 ∩ · · · ∩ Ak) ∩ Ak+1) U B ............Associative property for intersection   
  
Assume that (A1 ∩ A2 ∩ · · · ∩ Ak) = C   
  
=(A1 ∩ A2 ∩ · · · ∩ Ak+1) U B = ((A1 ∩ A2 ∩ · · · ∩ Ak) ∩ Ak+1) U B   
  
=(C ∩ Ak+1) U B   
  
= (C U B) ∩ (Ak+1 U B)by distributive property of union of intersection   
  
=((A1 ∩ A2 ∩ · · · ∩ Ak) U B) ∩ (Ak+1 U B)   
  
But (A1 ∩ A2 ∩ · · ∩ Ak) U B = (A1 ∪ B) ∩ (A2 ∪ B) ∩ · · ∩ (Ak ∪ B) The statement is true for k   
  
=((A1 ∩ A2 ∩ · · · ∩ Ak) U B) ∩ (Ak+1 U B) = (A1 ∪ B) ∩ (A2 ∪ B) ∩ · · ∩ (Ak ∪ B)∩ (Ak+1 U B)   
  
= The statement is true for k+1   
  
=By mathematical induction, the statement is true for any positive integer n.